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TO THE QUESTION ABOUT THE ABILITY OF SYNTHESIZING THE CRITERION OF WALD-SAVAGE

LABSKER LEV (ORCID 0000-0002-4143-9023)[1]

[1]Financial University under the Government of the Russian Federation

Abstract. In games with nature, as an optimality principle in adverse economic conditions, the edge-not pessimistic Wald payoff-criterion and the Savage risk-criterion are often used. In this paper, we introduce a linear convolution of these criteria, called the Wald-Savage synthetic criterion with a payoff-indicator $\alpha \in [0.1]$ expressing the quantitative ratio of the decision-maker to the payoffs. This criterion makes it possible to assess the optimality from the synthetic (joint) position of payoffs and risks. The conditions for the existence and uniqueness of the value of the payoff-indicator are found at which the Wald-Savage criterion has the property of synthesis consisting in the existence of a strategy optimal by the Wald-Savage criterion but not optimal for any of the constituent criteria.

Keywords: game with nature, the Wald criterion, the Savage criterion, the synthetic Wald-Savage criterion, payoff-indicator, synthesizing.

As you know, decision-making is the most important component of any management.

In the analysis of the task of making financial-economic decisions useful is the use of a model "Game with nature" [1, pp. 12-59], in which the principles of optimality of strategies by different criteria. Some of them are - the *payoffs-criteria* to determine optimality from the point of view of payoffs, abstracting from risk (for example, criteria of Wald [2; 1, pp. 273-308], maxymax [1, p. 349-362], etc.). Other – *risk-criteria*, on the contrary, characterize the optimality from the point of view of risks, without taking into account the obvious payoffs (for example, Savage criteria [3; 1, pp. 308-349], minimin [1, pp. 362-376], etc.). Widely used *combined* criteria, is composed of two payoff-criteria or two risk-criteria. In each such pair, one of the criteria is extremely pessimistic, and the other is extremely optimistic (for example, the Hurwicz payoff-criterion [4; 5; 1, pp. 479-558] and the Hurwicz risk-criterion [1, pp. 534-558]).

In our opinion, deserves attention the use of paired *synthetic* criteria, composed of the payoff-criteria and risk-criteria because they provide an opportunity to assess the optimality of the strategies with synthetic (joint) the point of view of payoffs and gambling risks. A general approach to the design of such criteria is proposed in [1], and in [6] and [7] a synthetic Wald-Savage criterion was introduced. For its description let us briefly recall the necessary definitions.

Let in the game with nature $S^p = \{A_1, A_2, ..., A_m\}$ [1], $m \geq 2$ – a set of alternative pure strategies of the player A; $\Pi_1, \Pi_2, ..., \Pi_n$, $n \geq 2$, – states of nature Π; real numbers $a_{ij}, i \in I = \{1,2,...,m\}$, $j \in J = \{1,2,...,n\}$, – the payoffs of the player A in a game situation (A_i, Π_j), when a player A selects the strategy A_i, and the nature is in a state of Π_j; $\beta_j = \max\{a_{ij} : i \in I\}$, $j \in J$, - an indicator of the favorability of the state of Π_j; $r_{ij} = \beta_j - a_{ij}$, $i \in I, j \in J$ – the risk of not receiving the player A when choosing the strategy A_i of the greatest win in the state of nature Π_j of the payoff β_j [1, pp. 18-25; 8, pp. 9-61].

According to the Wald criterion [2; 1, pp. 273-308; 8, pp. 73-94; 9, pp. 330-347]; $W_i = \min\{a_{ij} : j \in J\}$- the indicator of the effectiveness of the strategy A_i, $i \in I$;

[1] In the designation S^p, the letter "p"-the first letter of the *pure*, indicates that the strategies $A_1, A_2, \ldots, A_m$ considered in this article are *pure*, and not mixed, i.e. are chosen by player A in a *certain* way without admixtures of chance and uncertainty.

$W_{S^p} = \max\{W_i : i \in I\}$; – the price of the game in the set S^p; strategy A_k - optimal if $W_k = W_{S^p}$; $\left(S^p\right)^{O(W)}$ – the set of strategies that is optimal in the set S^p.

According to the criterion of Savage [3; 1, pp. 308-349; 8, pp. 95-120; 9, pp. 348-370]: $Sav_i = \max\{r_{ij} : j \in J\}$ - the inefficiency of the strategy A_i, $i \in I$; $Sav_{S^p} = \min\{Sav_i : i \in I\}$ – the price of the game in the set S^p; the strategy A_k is optimal if $Sav_k = Sav_{S^p}$; $\left(S^p\right)^{O(Sav)}$; - a set of strategies optimal in the set S^p.

Wald-Savage criterion with a payoff-indicator $\alpha \in [0,1]$ is determined by the following components:

$$(WSav)_i(\alpha) = \alpha W_i - (1-\alpha) Sav_i \tag{1}$$

- an indicator of the effectiveness of the strategy A_i, $i \in I$; the price of the game is determined by the formula

$$(WSav)_{S^p}(\alpha) = \max\{(WSav)_i(\alpha) : i \in I\}; \tag{2}$$

strategy A_k we call optimal if $(WSav)_k(\alpha) = (WSav)_{S^p}(\alpha)$; $\left(S^p\right)^{O((WSav)(\alpha))}$ – set of strategies optimal in the set S^p.

Payoff-indicator $\alpha \in [0,1]$ and risk-indicator $(1-\alpha) \in [0,1]$ are quantitative indicators of the degree of preference given to the player A accordingly, the gains and risks.

From (1) it is obvious that the graphs of the strategies efficiency indicators according to the Wald-Savage criterion as linear functions of the argument α are m segments. From (2) we conclude that the graph of the price of the game is the upper envelope of these segments, which is a broken line, the number of links l which does not exceed m.

Definition 1. We will say that in this game with the nature of the Wald-Savage criterion with a fixed value of the payoff-indicator $\alpha \in (0,1)$ *it has the property of synthesizing if there is an optimal strategy for this criterion, which is not optimal either by Wald's criterion or by Savage's criterion.*

In the article the necessary and sufficient conditions for the existence and uniqueness of the value of the win-indicator, in which the Wald-Savage criterion has the property of synthesizing, are found.

To formulate the corresponding theorems, we define the following sets of optimal strategies and the prices of the game in these sets: $((S^p)^{O(W)})^{O(Sav)}$ - a set of strategies that are optimal according to the criterion of Savage in the set $(S^P)^{O(W)}$; $Sav_{(S^p)^{O(W)}} = \min\{Sav_i : A_i \in (S^p)^{O(W)}\}$ - the price of the game by Savage criterion in the set $(S^P)^{O(W)}$; $((S^p)^{O(Sav)})^{O(W)}$ - a set of strategies that are optimal according to the criterion of Wald in set $(S^P)^{O(Sav)}$; $W_{(S^p)^{O(Sav)}} = \max\{W_i : A_i \in (S^p)^{O(Sav)}\}$ - the price of the game according to the criterion of Wald in the set $(S^p)^{O(Sav)}$.

Definition 2. We will say that playing with nature satisfies the condition $\sigma_{(WSav)}$, if for each strategy $A_i \notin (S^p)^{O(W)} \cup (S^p)^{O(Sav)}$ the following inequality holds

$$\begin{aligned}(Sav_{(S^p)^{O(w)}} - Sav_{S^p})W_i - \\ -(W_{S^p} - W_{(S^p)^{O(Sav)}})Sav_i \le \\ \le W_{(S^p)^{O(Sav)}} Sav_{(S^p)^{O(W)}} - W_{S^p} Sav_{S^p}\end{aligned} \tag{3}$$

and there is a strategy $A_k \notin (S^p)^{O(W)} \cup (S^p)^{O(Sav)}$, for which inequality (3) becomes equality. Let

$$\alpha_{(WSav)} = \frac{Sav_{(S^p)^{O(W)}} - Sav_{S^p}}{(Sav_{(S^p)^{O(W)}} - Sav_{S^p}) + (W_{S^p} - W_{(S^p)^{O(Sav)}})},$$

$$\delta_{(WSav)} = \frac{W_{(S^p)^{O(Sav)}} \cdot Sav_{(S^p)^{O(W)}} - W_{S^p} \cdot Sav_{S^p}}{(Sav_{(S^p)^{O(W)}} - Sav_{S^p}) + (W_{S^p} - W_{(S^P)^{O(Sav)}})}$$

and $S^p_{\alpha_{(WSav)}} = \{A_i : (WSav)_i(\alpha_{(WSav)}) = \delta_{(WSav)}\}$ - a set of strategies, the efficiency of which according to the Wald-Savage criterion at the payoff-indicator $\alpha_{(WSav)}$ equal $\delta_{(WSav)}$.

The following two theorems are valid provided that in the game with nature there is no strategy that is optimal at the same time by Wald's criterion and Savage's criterion, and there is a strategy that is not optimal neither by Wald's criterion nor by Savage's criterion.

Theorem 1 (necessary conditions).

If there is a single value of the payoff-indicator, in which the Wald-Savage criterion has the property of synthesizing, the following statements are true;

a) the number l of links in the broken line representing the price chart of the game according to the Wald-Savage criterion is 2;

b) the only value of the payoff-indicator, in which the Wald-Savage criterion has the property of synthesizing, is $\alpha_{(WSav)}$;

c) a set of strategies that are optimal according to the criterion of Wald-Savage, has the following structure:

$$(S^p)^{O((WSav)(\alpha))} =$$
$$= (S^p)^{O(Sav)}, \text{ when } \alpha = 0;$$
$$= ((S^p)^{O(Sav)})^{O(W)}, \text{ when } 0 < \alpha < \alpha_{(WSav)};$$
$$= ((S^p)^{O(Sav)})^{O(W)} \cup ((S^p)^{O(W)})^{O(Sav)} \cup S^p_{\alpha_{(WSav)}},$$
$$\text{when } \alpha = \alpha_{(WSav)};$$
$$= ((S^p)^{O(W)})^{O(Sav)}, \text{ when } \alpha_{(WSav)} < \alpha < 1;$$
$$= (S^p)^{O(W)}, \text{ при } \alpha = 1; \quad (4)$$

d) the condition $\sigma_{(WSav)}$ is satisfied.

Theorem 2 (sufficient conditions).

From each of the following two conditions a) let the set $S^p_{\alpha_{(WSav)}}$ not empty and set $(S^p)^{O((WSav)(\alpha))}$ strategies that are optimal according to the criterion of Wald-Savage, has the structure (4);

b) the condition $\sigma_{(WSav)}$ is satisfied, it follows that $\alpha_{(WSav)}$ is the only value of the payoff-indicator, when which Wald-Savage criterion has the property of synthesizing.

It follows from theorems 1 and 2 that the condition $\sigma_{(WSav)}$ it is necessary and sufficient for the existence and uniqueness of the value of the payoff-indicator, in which the Wald-Savage criterion has the property of synthesizing.

The presence of a set of values of the payoff-indicator, in which there are synthesized strategies, leads to uncertainty in the choice of the payoff-indicator. The obtained results exclude a specified uncertainty, giving rise to another possible uncertainty in the choice of strategy from among the several synthetic strategies that might exist in this case only the value of the prize increased.

The obtained results are applicable to the analysis of any problem of financial and economic decision-making in the conditions of uncertainty, allowing the use of the “Game with nature” model with a synthetic criterion of Wald-Savage optimality.

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