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DISPERSION OF THE CAPACITY METHOD FROM THE POSITION IN THE DISTRIBUTORS CHAIN

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Abstract. The rare events analysis is an actual problem. The capacity method in trading is capable of recovering the original regularities of rare sales with great accuracy. However, when distancing from the end consumer in the chain of distributors, the accuracy decreases. In the previous study, when modeling the consumption process, it was noted that the precision falls, but falls not significantly. In the current study, the reason for the falls in precision was explained and it was shown that the error grows as a decreasing geometric progression. Also, the variance and the standard deviation are determined for the error. It is shown that the dispersion and the mean square deviation grows even more slowly than the geometrically decreasing progression. Numerical error values are obtained depending on the position in the chain of distributors.

Keywords: capacity method, accuracy, error, sequence of distributors, intermediaries.

Rare events, which are represented not as a time series, but as a set of data about the time of an event and the magnitude of this event, for example, data on rare sales, are well analyzed using a capacity method [1, 2], which restores the dependence of the products consumption rate with time or any other dependence that leads to the event occurrence.

For distributors, that working directly with end-users, the error of restoring the initial dependence is only 1-3%. Knowing the speed with which the product is spent, you can easily predict the moment when the buyer will need the next purchase and use it for your own purposes to optimize profit or expenses.

However, while distancing from end-users, when intermediaries appear in the distributors chain, the precision falls. According to the previous study [3, 4], as a result of the simulation, the precision falls was as follows, as shown in the Table 1.

Table 1

Precision falls, depending on the position in the distributor chain

	1 intermediary	2 intermediaries	3 intermediaries
The average value of the relative deviation for 20 runs, %	4,2889	5,6544	6,0386
The root of the sample variance of the relative deviation for 20 runs, %	1,0284	1,3967	1,2145

Source: The results of the simulation were obtained by the author

A detailed analysis of the consumption process and restoration of the initial dependence showed that the error arises from the fact that there is a discrepancy ΔQ ΔQ between the observed and consumed output over the period of time between purchases. This happens due to the fact that part of the purchases related to the previous period of time, and also because you have to use insurance stocks, see Fig. №1,

$$\Delta Q_S = \sum_j \left(S_{k+1}^j - S_k^j\right)$$

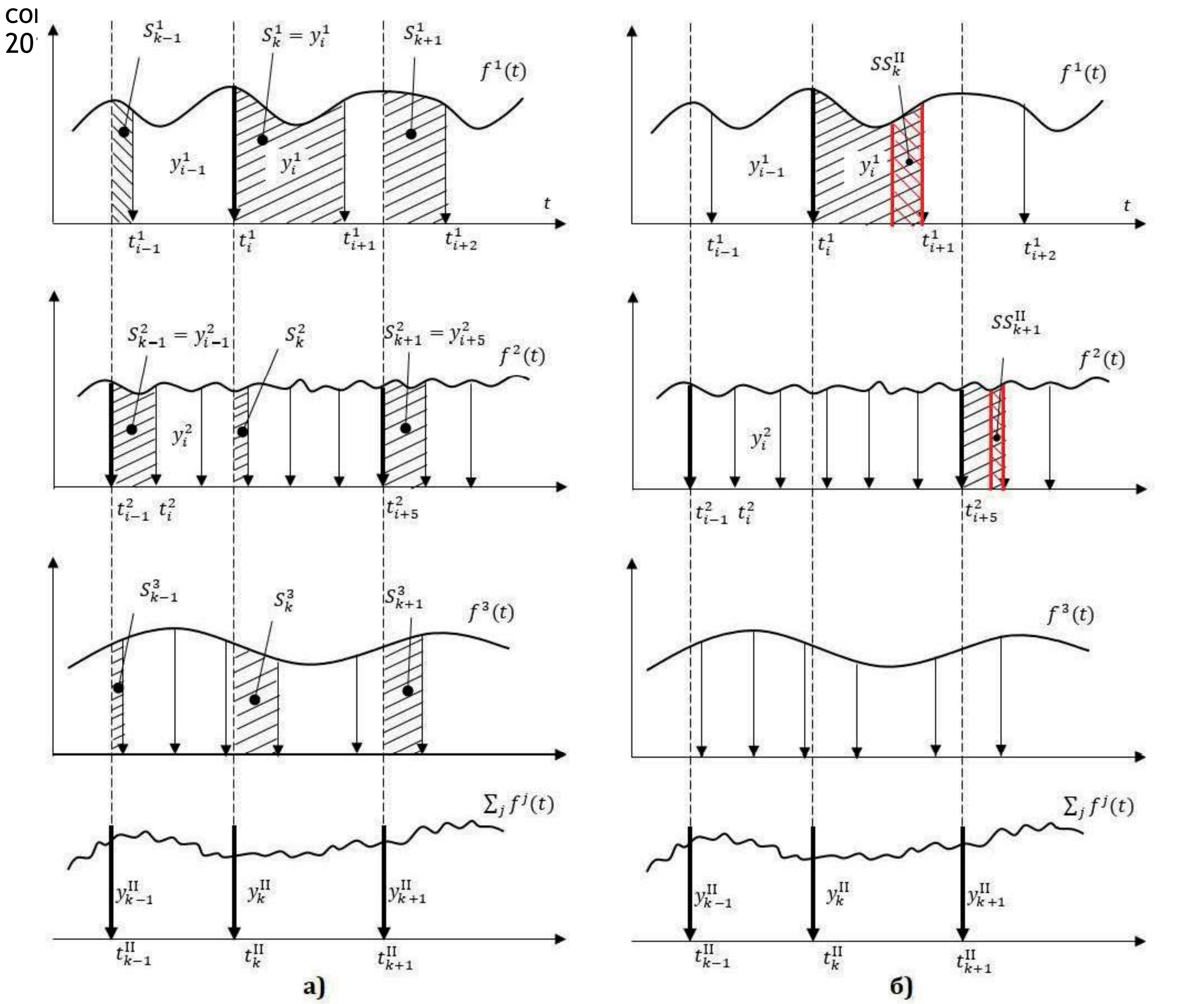

Fig. 1 The discrepancy between the observed and consumed output: a) part of the purchase related to previous period; b) use of insurance stocks
Source: obtained by the author

$$\Delta Q_{SS} = SS^{II}_{k+1} - SS^{II}_k$$

$$\Delta Q = \Delta Q_S - \Delta Q_{SS}$$

$$S^j_k \in (0; y^j_{\text{предш}.k}]$$

$$SS^{II}_k \in (0; \max_j y^j_{\text{предш}.k}]$$

where the value of $y^j_{\text{предш}.k}$ indicates the volume of the previous (inclusive) purchase of the buyer j with respect to the time t^{II}_k.

If a second level of intermediaries is added, the magnitude of the discrepancy will include inconsistencies from smaller intermediate distributors.

$$\Delta Q^{II}_S = \Sigma_r\left(S^{r,II}_{L+1} - S^{r,II}_L\right) + \Sigma_r \Sigma_j (S^{r,j}_{L+1} - S^{r,j}_L)$$

$$\Delta Q^{II}_{SS} = SS^{III}_{L+1} - SS^{III}_L + \Sigma_r (SS^{r,II}_{L+1} - SS^{r,II}_L)$$

For three levels of intermediaries, the magnitude of the discrepancy will also include inconsistencies from new levels.

The stock of products of the top distributors is always several times larger than the stock of products of smaller distributors. If we consider the idealized case, when the distributors have the same number n of child distributors, as well as the stocks of the upper distributors is always enough for P purchases of child distributors, then the magnitude of the discrepancy may be expressed as follows:

$$\Delta Q_S^{III} = n \cdot P^2 \cdot dS + n^2 \cdot P \cdot dS + n^3 \cdot dS$$

$$\Delta Q_{SS}^{III} = P^2 \cdot dSS + n \cdot P \cdot dSS + n^2 \cdot dSS$$

where

$$dS = S_{k+1}^j - S_k^j, \quad dSS^{\text{II}} = SS_{k+1}^{\text{II}} - SS_k^{\text{II}}.$$

A general discrepancy then will be

$$\Delta Q^{III} = \Delta Q_S^{III} - \Delta Q_{SS}^{III} =$$

$$= (n \cdot dS - dSS)(P^2 + nP + n^2)$$

The relative error can be obtained if the magnitude of the discrepancy is divided by the amount of the purchase taking place at a given position in the distributors chain.

$$\frac{\Delta Q^{III}}{y^{\text{IV}}} = \frac{(n \cdot dS - dSS)}{P \cdot y_i^j}\left(1 + \frac{n}{P} + \left(\frac{n}{P}\right)^2\right)$$

If we continue this sequence further, we obtain a relative error for NN levels of intermediaries.

$$\frac{\Delta Q^{[N]}}{y^{[N+1]}} = \frac{(n \cdot dS - dSS)}{P \cdot y_i^j}\left(1 + \frac{n}{P} + \left(\frac{n}{P}\right)^2 + \cdots + \left(\frac{n}{P}\right)^{N-1}\right)$$

The expression in parentheses is a geometrically decreasing progression, on the condition that the value $\left(\frac{n}{P}\right)$ is less than 1, which is usually satisfied, since n shows the number of customers, and P shows how many purchases the distributor's stock satisfy.

There are reasons to assume that the value of $S_\cdot^j$ is distributed uniformly over the interval $(0; y_{\text{предш}.k}^j]$, and SS_k^{II} is independent of it along the interval $(0; \max_j y_{\text{предш}.k}^j]$ (in the idealized case for the same interval $(0; y_{\text{предш}.k}^j]$).

Now we can calculate the variance for the relative error. At the first level, there are $2(n+1)$ random variables involved (2 values per dS and dSS). On the second, $2(n^2+n)$, and on the third $2(n^3+n^2)$ random variables.

Since the ratio of the random variable $S_k^j \in (0; y_{\text{предш}.k}^j]$ to $y_{\text{предш}.k}^j$ gives a base uniform random variable from 0 to 1 with a variance of 1/12, we obtain an expression for the variance:

$$D\left[\frac{\Delta Q^{[N]}}{y^{[N+1]}}\right] = \frac{2(n+1)}{12P^2} + \frac{2(n^2+n)}{12P^4} + \frac{2(n^3+n^2)}{12P^6} + \cdots + \frac{2(n+1)n^{N-1}}{12P^{2N}} =$$

$$= \frac{n+1}{6P^2}\left(1 + \frac{n^1}{P^2} + \frac{n^2}{(P^2)^2} + \cdots + \frac{n^{N-1}}{(P^2)^{N-1}}\right)$$

The expression in parentheses is again the sum of a geometrically decreasing progression, which decreases even faster. The sum of the first N terms is $\frac{1-q^N}{1-q}$, where $q = n/P^2$ is the denominator of the progression.

$$D\left[\frac{\Delta Q^{[N]}}{y^{[N+1]}}\right] = \frac{(n+1)\left(1 - \left(\frac{n}{P^2}\right)^N\right)}{6P^2\left(1 - \frac{n}{P^2}\right)}$$

The mean square deviation is obtained as the root of the variance:

$$\sigma\left[\frac{\Delta Q_S}{y^{[N+1]}}\right] = \sqrt{\frac{(n+1)\left(1 - \left(\frac{n}{P^2}\right)^N\right)}{6P^2\left(1 - \frac{n}{P^2}\right)}}$$

In conclusion, lets represent a table in which the variance and the mean square deviation of the relative discrepancy between the observed and consumed output are obtained for the time between two purchases from the distance from the end-users under idealized conditions, Table 2.

As can be seen from the table, the position in the distributor chain practically does not affect the precision with which the original pattern, that leading to the occurrence of purchases (rare events), will be restored.

Table 2

Dispersion and standard deviation of relative error when moving away from the end user

N	$n = 3n = 3$, $P = 10P = 10$		$n = 3n = 3$, $P = 4P = 4$		$n = 20n = 20$, $P = 20P = 20$	
	D	$\sigma\sigma$, %	D	$\sigma\sigma$, %	D	$\sigma\sigma$, %
1	0,006666667	8.164966	0,041666667	20.412415	0,00875	9.3541435
2	0,006866667	8.286535	0,049479167	22.243910	0,0091875	9.5851448
3	0,006872667	8.290155	0,05094401	22.570780	0,009209375	9.5965489
4	0,006872847	8.290263	0,051218669	22.631542	0,009210469	9.5971187
5	0,006872852	8.290267	0,051270167	22.642917	0,009210523	9.5971472

Source: obtained by the author

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